Geographically Weighted Functional Regression Analysis

Yoshihiro Yamanishi
Okayama University, Graduate School of Natural Science & Technology
3-1-1 Tsushima-Naka
Okayama 700-8530, Japan
yoshi@ems.okayama-u.ac.jp

Yutaka Tanaka
Okayama University, Department of Environmental and Mathematical Sciences
3-1-1 Tsushima-Naka
Okayama 700-8530, Japan
tanaka@ems.okayama-u.ac.jp

1. Introduction and Summary

Functional regression analysis has been studied so far by Ramsay and Silverman (1997). This method enables us to investigate the relationship among the variables over time. Sometimes, however, we meet the case where regression coefficients do not remain fixed over space, especially in analyzing the spatial data. Present paper proposes geographically weighted functional regression analysis to analyze the relationship among variables which varies over space as well as over time, borrowing the idea of Brunsdon and et.al (1998) in which geographical weight is considered in ordinary regression. A procedure is proposed for estimating the regression coefficient function, and the bootstrap is applied to evaluate the reliability of the estimated weight functions.

2. Functional Regression Analysis

To predict a functional response $y(t)$ from some functional covariates $x_g(s)$, we consider a functional multiple regression model as follows:

$$y_i(t) = \beta_0(t) + \sum_{g=1}^{G} \int x_{ig}(s)\beta_g(s,t)ds + \epsilon_i(t) \qquad (i = 1, \cdots, N), \tag{1}$$

where $\beta_0(t)$ is a mean function, G is the number of functional covariates, $\beta_g(s,t)$ is a regression coefficient function, $\epsilon_i(t)$ is a random error function and N is the number of observations.

3. Geographically Weighted Functional Regression Analysis

To deal with the spatial non-stationarity of the regression coefficient functions, we propose a geographically weighted functional multiple regression model as follows:

$$y_i(t) = \beta_0(t) + \sum_{g=1}^{G} \int x_{ig}(s)\beta_g(s,t,p_i)ds + \epsilon_i(t) \qquad (i = 1, \cdots, N), \tag{2}$$

where p_i means the geographical location of the i-th observation, and $\beta_g(s,t,p_i)$ is a regression coefficient function. Considering the distance d_{ik} between location i and location k $(k = 1, \cdots, N)$ in predicting the i-th functional responce, the geographical weight defined as $\alpha_{ik} = \exp(-d_{ik}/h)$ is introduced into the procedure of estimating $\beta_g(s,t)$. As a measure of the spatial variation for the relationship between the variables over time, we propose a statistic to assess the variability of $\beta_g(s,t,p_i)$ as i varies for a fixed g. Our statistic is the following integrated variance of $\beta_g(s,t,p_i)$ across i:

$$v_g = \frac{1}{N}\sum_{i=1}^{N} \int\int \{(\beta_g(s,t,p_i) - \beta_g(s,t,\cdot))\}^2 dtds \quad (g = 1, \cdots, G) \tag{3}$$

where a dot denotes averaging over subscript i. This implies that the higher the value of v_g, the stronger the evidence that the regression coefficient $\beta_g(s,t)$ has a large spatial variation.

REFERENCES

Brunsdon, C., Fotheringham, S. and Charlton, M. (1998). Geographically weighted regression - modelling spatial non-stationarity, *J. Royal Statist. Soc., 47, 3, 431-443.*

Ramsay, J. O. and Silverman, B.W. (1997). *Functional Data Analysis, Springer.*

Shimokawa, M., Mizuta, M. and Sato, Y. (2000). An Expansion of Functional Regression Analysis (in Japanese), *Japanese Journal of Applied Statistics, 29-1, 27-39.*

RÉSUMÉ

L'analyse fonctionnelle de régression nous permet d'étudier le rapport parmi les variables avec le temps. Parfois, cependant, nous rencontrons le cas òu les coefficients de régression ne demeurent pas fixes au-dessus de l'espace, particulièrement en analysant les donnèes spatiales. Le papier actuel propose l'analyse fonctionnelle géographiquement pesée de régression pour analyser le rapport parmi des variables qui change au-dessus de l'espace aussi bien que le temps fini.