Learning from rankings

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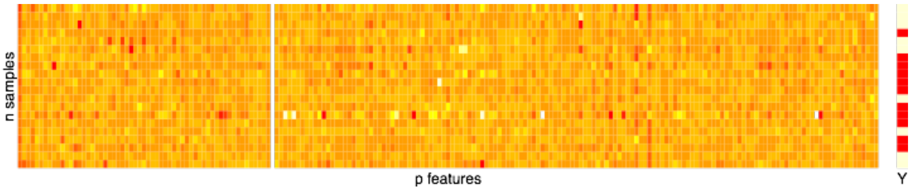
Nonparametric Methods for Large Scale Representation Learning
NIPS workshop, Montreal, Dec 11, 2015

Motivation

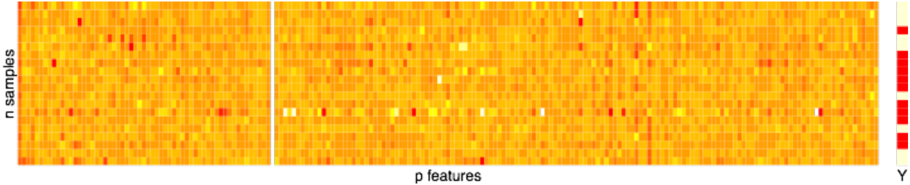

Patients with same condition
DNA
Profiling
Good responders
No Responders
Bad side effects

Machine learning formulation



- Challenges
  - $n << p$
  - noisy data, subject to various technical variations
- Standard approach:
  1. Normalize data (difficult)
  2. Learn in high dimension with normalized data
- Skip normalization issue
  1. Find a simpler, more robust representation
  2. Learn with the simpler representation

Machine learning formulation



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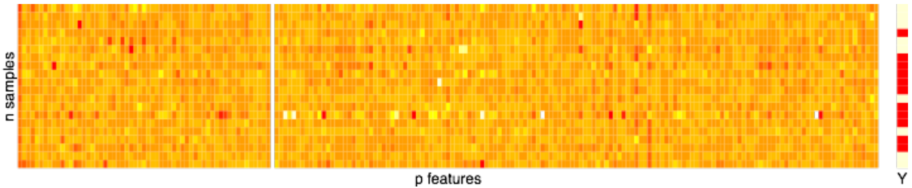

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Outline

1. SUQUAN: Supervised full quantile normalization (w. M. Le Morvan)
2. The Kendall and Mallows kernels (w. Y. Jiao)

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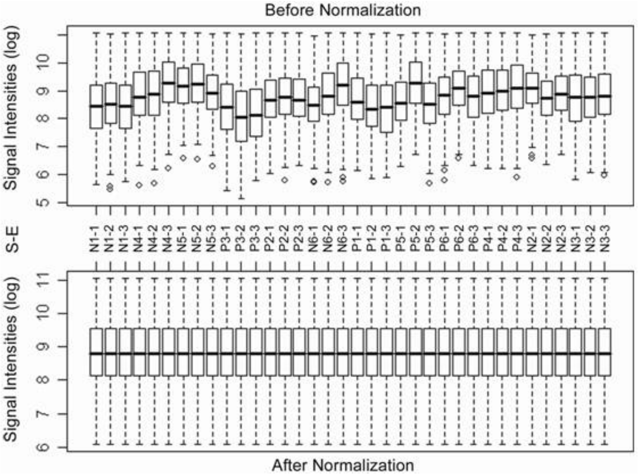
Full quantile normalization



How to choose the target distributions?
Gaussian? Uniform? CDF of the data?

Learning the target distribution

- Let $f \in \mathbb{R}^p$ a non-decreasing target distribution (CDF)
- For $x \in \mathbb{R}^p$, let $\Phi_f(x) \in \mathbb{R}^p$ be the data after full quantile normalization with target distribution f
- Learn a (generalized) linear model over normalized data:

$$\min_{w,b} \frac{1}{n} \sum_{i=1}^{n} \ell\left(w^\top \Phi_f(x_i) + b\right) + \lambda \Omega(w)$$

- SUQUAN: jointly learn f and (w, b):

$$\min_{w,b,f} \frac{1}{n} \sum_{i=1}^{n} \ell\left(w^\top \Phi_f(x_i) + b\right) + \lambda \Omega(w)$$

SUQAN: supervised quantile normalization

- For $x \in \mathbb{R}^p$, let $\Pi_x \in \mathbb{R}^{p \times p}$ the permutation matrix of x's entries
- Quantile normalized x with target distribution f is:

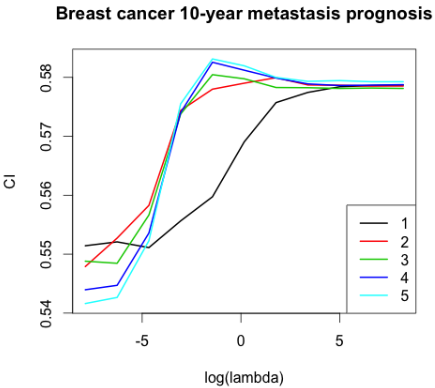
$$\Phi_f(x) = \Pi_x f$$

- SUQUAN solves

$$\begin{aligned} & \min_{w,b,f} \frac{1}{n} \sum_{i=1}^{n} \ell \left(w^\top \Pi_x f + b \right) + \lambda \Omega(w) \\ = & \min_{w,b,f} \frac{1}{n} \sum_{i=1}^{n} \ell \left(< wf^\top, \Pi_x > + b \right) + \lambda \Omega(w) \end{aligned} \quad (1)$$

- A particular rank-1 matrix optimization
- Efficiently solved by alternatively optimizing f (isotonic GLM) and w

Results (preliminary)



Breast cancer prognosis from gene expression data (survival logistic regression)

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1. SUQUAN: Supervised full quantile normalization (w. M. Le Morvan)
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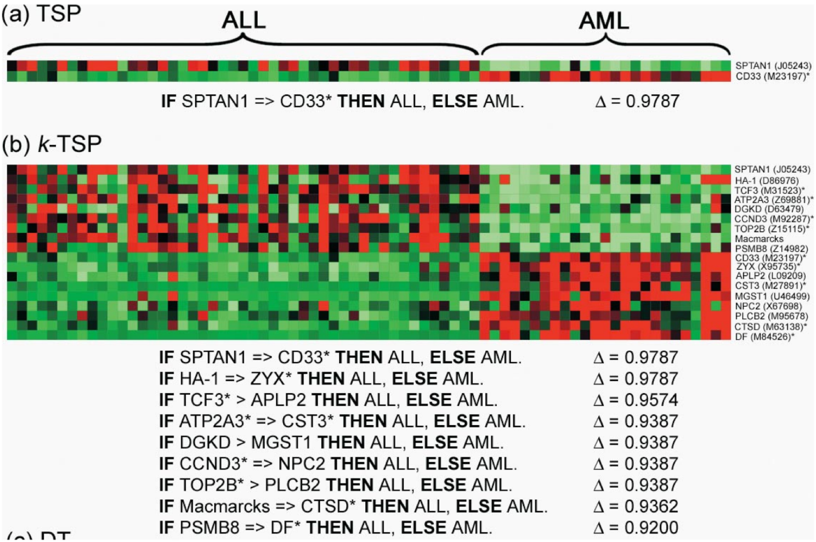
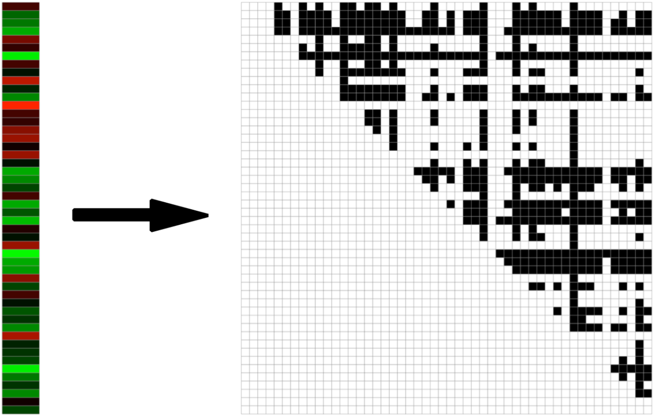
An idea: Top scoring pairs (TSP)



(Geman et al., 2004; Tan et al., 2005; Leek, 2009)

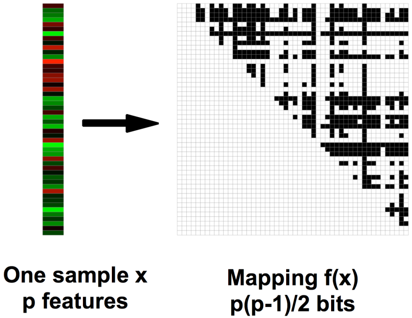
More generally: all pairwise comparisons



One sample x
p features

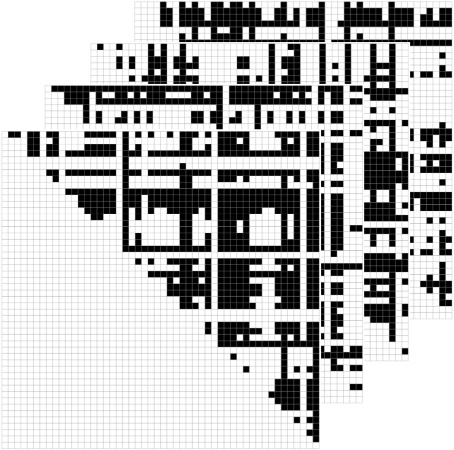
Mapping f(x)
p(p-1)/2 bits

Remark: representation of the symmetric group



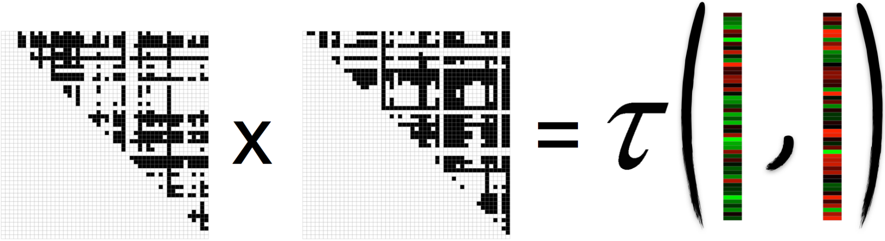
- Obviously, this representation as $O(p^2)$ bits exists for any ranking or permutation of p items
- Many other applications in learning over rankings, learning to rank, learning permutations etc...
- We are interested particularly in practical solutions when p is large

Practical challenge



- Need to store $O(p^2)$ bits per sample
- Need to train a model in $O(p^2)$ dimensions

The good old kernel trick



O(p^2)

O(p log(p))

More formally

- For two permutations σ, σ' let $n_c(\sigma, \sigma')$ (resp. $n_d(\sigma, \sigma')$) the number of concordant (resp. discordant) pairs.
- The Kendall kernel (a.k.a. Kendall tau coefficient) is defined as

$$K_\tau(\sigma, \sigma') = \frac{n_c(\sigma, \sigma') - n_d(\sigma, \sigma')}{\binom{p}{2}}.$$

- The Mallows kernel is defined for any $\lambda \geq 0$ by

$$K_M^\lambda(\sigma, \sigma') = e^{-\lambda n_d(\sigma, \sigma')}.$$

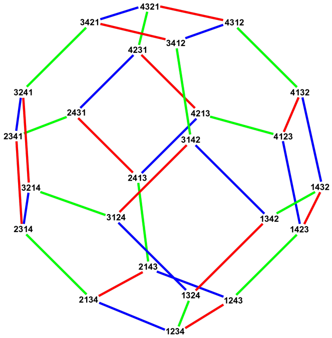
Theorem (Jiao and V., 2015)

The Kendall and Mallows kernels are positive definite.

Theorem (Knight, 1966)

These two kernels for permutations can be evaluated in $O(p \log p)$ time.

Related work



Cayley graph of $\mathbb{S}_4$

- Kondor and Barbarosa (2010) proposed the diffusion kernel on the Cayley graph of the symmetric group generated by adjacent transpositions.
- Computationally intensive ($O(p^p)$)
- Mallows kernel is written as

$$K_M^\lambda(\sigma, \sigma') = e^{-\lambda n_d(\sigma, \sigma')},$$

  where $n_d(\sigma, \sigma')$ is the shortest path distance on the Cayley graph.
- It can be computed in $O(p \log p)$

Application: supervised classification

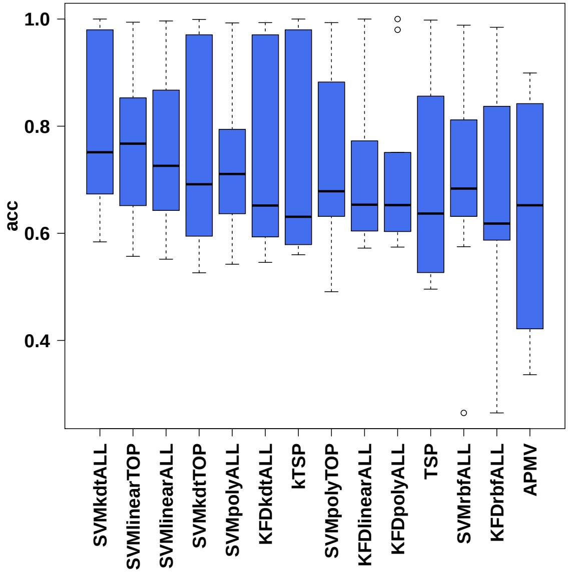
Datasets

Dataset	No. of features	No. of samples (training/test)	
		C_1	C_2
Breast Cancer 1	23624	44/7 (Non-relapse)	32/12 (Relapse)
Breast Cancer 2	22283	142 (Non-relapse)	56 (Relapse)
Breast Cancer 3	22283	71 (Poor Prognosis)	138 (Good Prognosis)
Colon Tumor	2000	40 (Tumor)	22 (Normal)
Lung Cancer 1	7129	24 (Poor Prognosis)	62 (Good Prognosis)
Lung Cancer 2	12533	16/134 (ADCA)	16/15 (MPM)
Medulloblastoma	7129	39 (Failure)	21 (Survivor)
Ovarian Cancer	15154	162 (Cancer)	91 (Normal)
Prostate Cancer 1	12600	50/9 (Normal)	52/25 (Tumor)
Prostate Cancer 2	12600	13 (Non-relapse)	8 (Relapse)

Methods

- Kernel machines Support Vector Machines (SVM) and Kernel Fisher Discriminant (KFD) with Kendall kernel, linear kernel, Gaussian RBF kernel, polynomial kernel.
- Top Scoring Pairs (TSP) classifiers [**?**].
- Hybrid scheme of SVM + TSP feature selection algorithm.

Results



Kendall kernel SVM

- **Competitive accuracy!**
- Less sensitive to regularization parameter!
- No need for feature selection!

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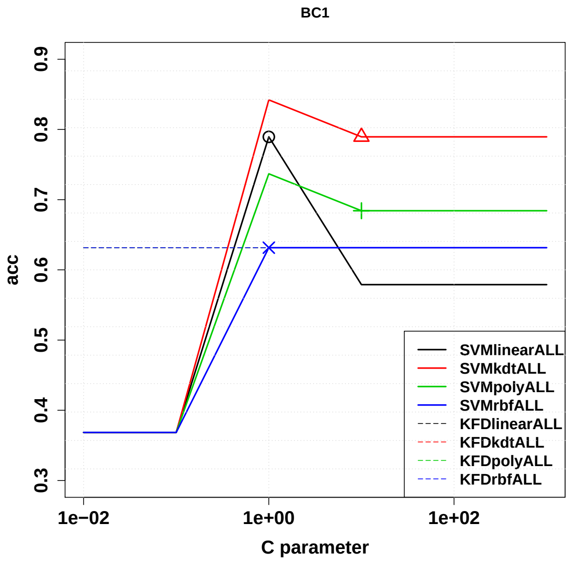

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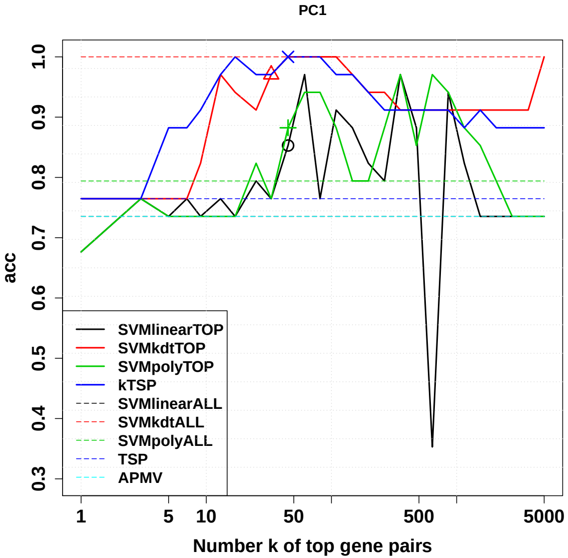

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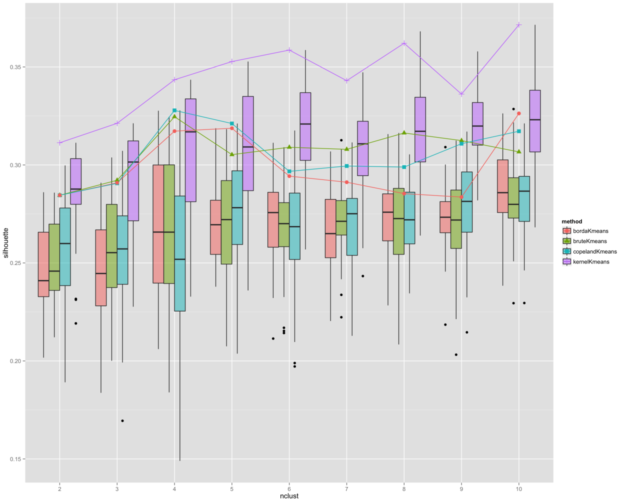
Application: clustering



- APA data (full rankings)
- $n = 5738$, $p = 5$
- (new) Kernel k-means vs (standard) k-means in $\mathbb{S}_5$
- Show silhouette as a function of number of clusters (higher better)

Extension to partial rankings

- Two interesting types of partial rankings are interleaving partial ranking

$$x_{i_1} \succ x_{i_2} \succ \cdots \succ x_{i_k}, \quad k \leq n.$$

and top-k partial ranking

$$x_{i_1} \succ x_{i_2} \succ \cdots \succ x_{i_k} \succ X_{\text{rest}}, \quad k \leq n.$$

- Partial rankings can be uniquely represented by a set of permutations compatible with all the observed partial orders.

Theorem

For these two particular types of partial rankings, the convolution kernel (Haussler, 1999) induced by Kendall kernel

$$K_\tau^\star(R, R') = \frac{1}{|R||R'|} \sum_{\sigma \in R} \sum_{\sigma' \in R'} K_\tau(\sigma, \sigma')$$

can be evaluated in $O(k \log k)$ time.

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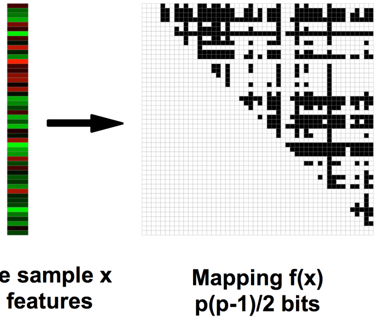
Extension to smoother, continuous representations

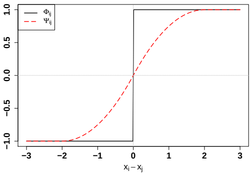


- Instead of $\Phi : \mathbb{R}^p \to \{0,1\}^{p(p-1)/2}$, consider the continuous mapping $\Psi_a : \mathbb{R}^p \to \mathbb{R}^{p(p-1)/2}$:

$$\Psi_a(x) = \mathbb{E}\Phi(x+\epsilon) \quad \text{with} \quad \epsilon \sim (\mathcal{U}[-\frac{a}{2}, \frac{a}{2}])^n$$

- Corresponding kernel $G_a(x, x') = \Psi_a(x)^\top \Psi_a(x')$

Computation of $G(x, x')$



- $G_a(x, x')$ can be computed exactly in $O(p^2)$ by explicit computation of $\Psi_a(x)$ in $\mathbb{R}^{p(p-1)/2}$

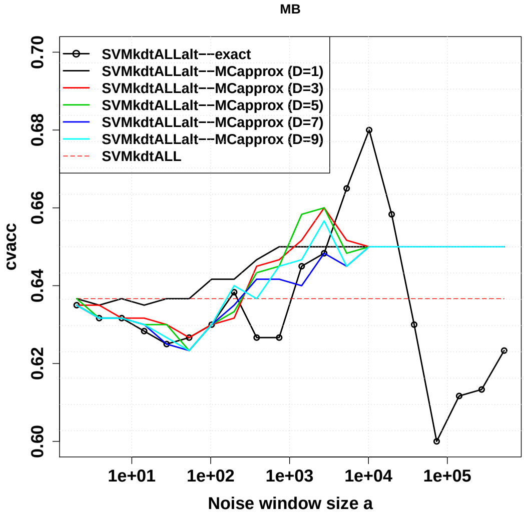
- $G_a(x, x')$ can be computed approximately in $O(D^2 p \log p)$ by Monte-Carlo approximation:

$$\tilde{G}_a(x, x') = \frac{1}{D^2} \sum_{i,j=1}^{D} K(x + \epsilon_i, x' + \epsilon_j')$$

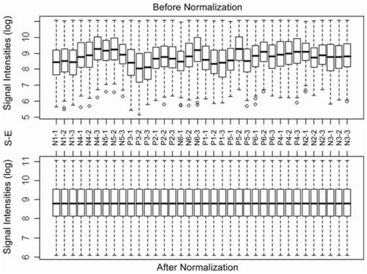
- Theorem: for supervised learning, Monte-Carlo approximation is better[1] than exact computation when $n = o(p^{1/3})$

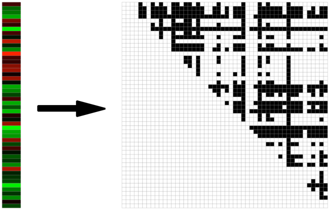
[1] faster for the same accuracy

Performance of $G_a(x, x)$



Conclusion





One sample x
p features

Mapping f(x)
p(p-1)/2 bits

- Full quantile normalization as matrix learning
- A representation of vectors that only depends on the relative order of features
- A tractable $O(p \log p)$ kernel for (partial) ranking and permutations
- Open questions
  - higher-order comparisons
  - primal approximation in less than $O(p^2)$ dimension
  - other applications (learning to rank etc..)

Thanks

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SEVENTH FRAMEWORK PROGRAMME

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MINES ParisTech

institutCurie
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