

# MVA "Kernel methods"

## Homework 4

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### Exercise 1.

Show that

$$\forall x, y \in \mathbb{R}, \quad K(x, y) = \max(0, 1 - |x - y|)$$

is a positive definite kernel on  $\mathbb{R}$ , and describe its RKHS.

### Exercise 2.

a. Describe the functions  $\phi : \mathbb{R}^+ \mapsto \mathbb{R}$  such that:

$$K(x, y) = \phi(\max(x, y))$$

is a positive definite kernel on  $\mathbb{R}^+$ .

b. Describe the functions  $\phi : \mathbb{R}^+ \mapsto \mathbb{R}$  such that:

$$K(x, y) = \phi(\min(x, y))$$

is a positive definite kernel on  $\mathbb{R}^+$ .

### Exercise 3.

Given two positive definite kernels  $K_1$  and  $K_2$  on a space  $\mathcal{X}$ , with respective RKHS  $\mathcal{H}_1$  and  $\mathcal{H}_2$ , and two positive scalars  $\alpha, \beta > 0$ , what is the RKHS of  $\alpha K_1 + \beta K_2$ ?