MVA "Kernel methods"
Homework 1

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Exercice 1. RKHS of the polynomial kernel

Describe the RKHS of the polynomial kernel

$$\forall \left(\mathbf{x}, \mathbf{x}'\right) \in \mathbb{R}^p, \quad K\left(\mathbf{x}, \mathbf{x}'\right) = \left(\langle \mathbf{x}, \mathbf{y} \rangle_{\mathbb{R}^p} + 1\right)^2$$

Exercice 2. Combining kernels.

1. Let K_1 and K_2 be two positive definite (p.d.) kernels on a set $\mathcal{X}$. Show that the functions $K_1 + K_2$ and $K_1 \times K_2$ are also p.d. on $\mathcal{X}$.

2. Let $(K_i)_{i\geq 1}$ a sequence of p.d. kernel on a set $\mathcal{X}$ such that, for any $(x, y) \in \mathcal{X}^2$, the sequence $(K_i(x, y))_{i\geq 0}$ be convergent. Show that the pointwise limit:

$$K(x, y) = \lim_{i\to+\infty} K_i(x, y)$$

is also p.d. (assuming the limit exists for any x, y).

Exercice 3. Quizz

Which of the following are p.d. kernels?

1. $\mathcal{X} = (-1, 1), \quad K\left(\mathbf{x}, \mathbf{x}'\right) = \frac{1}{1-\mathbf{x}\mathbf{x}'}$
2. $\mathcal{X} = \mathbb{N}, \quad K\left(\mathbf{x}, \mathbf{x}'\right) = 2^{\mathbf{x}\mathbf{x}'}$
3. $\mathcal{X} = \mathbb{R}_+, \quad K\left(\mathbf{x}, \mathbf{x}'\right) = \log\left(1 + \mathbf{x}\mathbf{x}'\right)$

4. $\mathcal{X} = \mathbb{R}, \quad K(\mathbf{x}, \mathbf{x}') = \exp\left(-|\mathbf{x} - \mathbf{x}'|^2\right)$
5. $\mathcal{X} = \mathbb{R}, \quad K(\mathbf{x}, \mathbf{x}') = \cos(\mathbf{x} + \mathbf{x}')$
6. $\mathcal{X} = \mathbb{R}, \quad K(\mathbf{x}, \mathbf{x}') = \cos(\mathbf{x} - \mathbf{x}')$
7. $\mathcal{X} = \mathbb{R}_+, \quad K(\mathbf{x}, \mathbf{x}') = \min(\mathbf{x}, \mathbf{x}')$
8. $\mathcal{X} = \mathbb{R}_+, \quad K(\mathbf{x}, \mathbf{x}') = \max(\mathbf{x}, \mathbf{x}')$
9. $\mathcal{X} = \mathbb{R}_+, \quad K(\mathbf{x}, \mathbf{x}') = \min(\mathbf{x}, \mathbf{x}')/\max(\mathbf{x}, \mathbf{x}')$
10. $\mathcal{X} = \mathbb{N}, \quad K(\mathbf{x}, \mathbf{x}') = GCD(\mathbf{x}, \mathbf{x}')$
11. $\mathcal{X} = \mathbb{N}, \quad K(\mathbf{x}, \mathbf{x}') = LCM(\mathbf{x}, \mathbf{x}')$
12. $\mathcal{X} = \mathbb{N}, \quad K(\mathbf{x}, \mathbf{x}') = GCD(\mathbf{x}, \mathbf{x}')/LCM(\mathbf{x}, \mathbf{x}')$

Note: bonus points if your proofs are particularly elegant or unique!