MVA "Kernel methods"
Homework 1

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Exercice 1.

1. Let K_1 and K_2 be two positive definite (p.d.) kernels on a set $\mathcal{X}$. Show that the functions $K_1 + K_2$ and $K_1 \times K_2$ are also p.d. on $\mathcal{X}$.

2. Let $(K_i)_{i \geq 1}$ a sequence of p.d. kernel on a set $\mathcal{X}$ such that, for any $(x, y) \in \mathcal{X}^2$, the sequence $(K_i(x, y))_{i \geq 0}$ be convergent. Show that the pointwise limit:

$$K(x, y) = \lim_{i \to +\infty} K_i(x, y)$$

is also p.d. (assuming the limit exists for any x, y).

3. Show that the following kernel is p.d.:

$$\forall -1 < x, y < 1 \quad K_1(x, y) = \frac{1}{1 - xy}.$$

Exercice 2.

Let Ω be a finite set of cardinality $|\Omega| = n$. Show that the following kernel defined on the set of subsets of Ω is p.d.:

$$\forall A, B \subset \Omega \quad K(A, B) = \frac{|A \cap B|}{|A \cup B|}.$$

Exercice 3.

Show that the kernel $K(x, y) = \min(x, y)$ is p.d. on $[0, 1]^2$. Describe its RKHS.

Exercice 4.

Prove that if $K : \mathcal{X} \times \mathcal{X}$ is a positive definite function, then it is the r.k. of a unique RKHS. (Hint: consider the linear space spanned by the functions $K_x : t \mapsto K(x,t)$, and use the fact that a linear subspace $\mathcal{F}$ of a Hilbert space $\mathcal{H}$ is dense in $\mathcal{H}$ if and only 0 is the only vector orthgonal to all vectors in $\mathcal{F}$)