

# Inferring and using biological networks

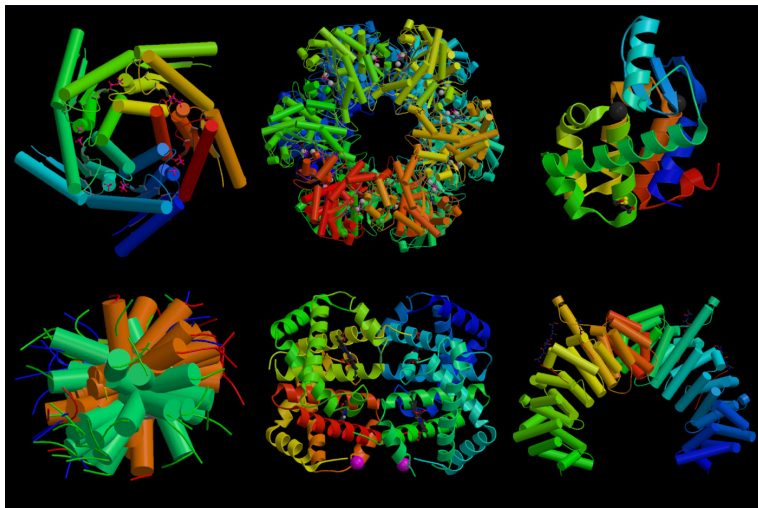
Jean-Philippe Vert

Jean-Philippe.Vert@mines-paristech.fr

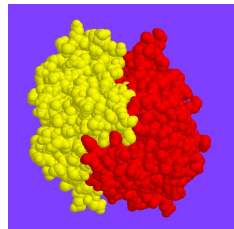
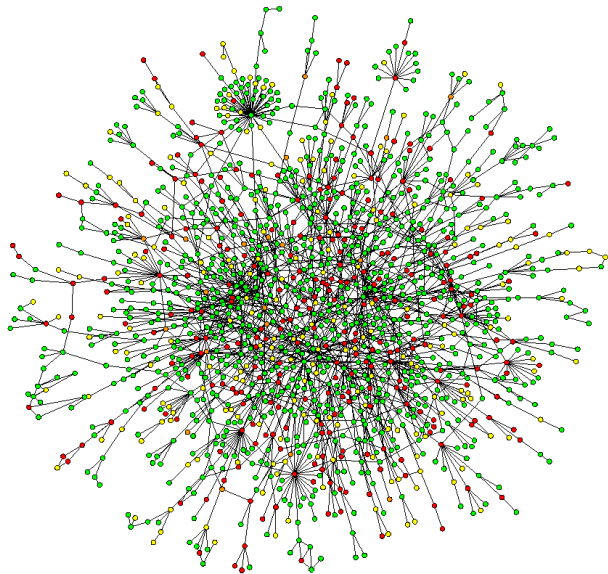
Mines ParisTech / Institut Curie / INSERM U900

Human Genome Center, Institute of Medical Science, University of  
Tokyo, August 4, 2009.

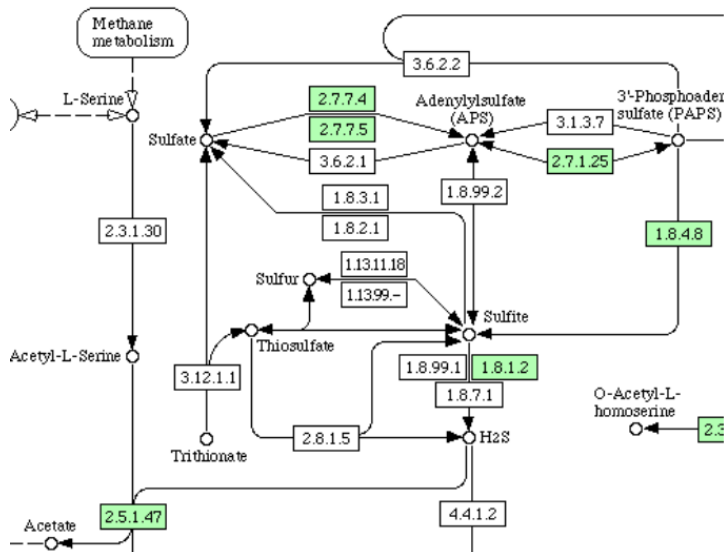
# We have many genes and proteins..



# Network 1: protein-protein interaction

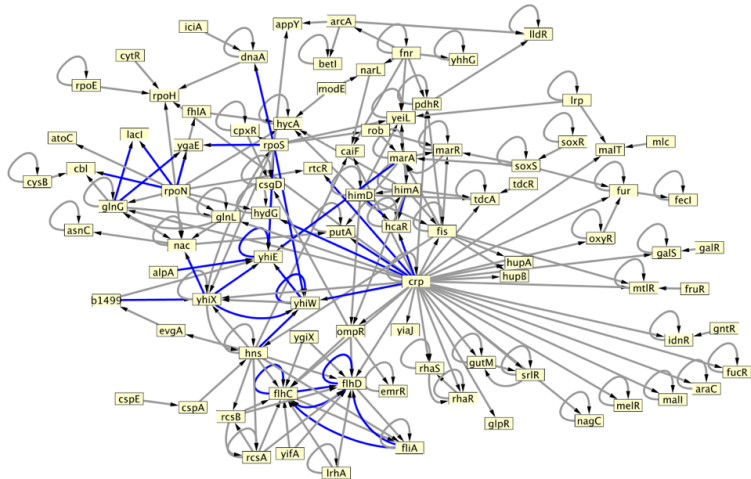


# Network 2: metabolic network





# Network 3: gene transcriptional regulatory network



*Biologists* have collected a lot of data about proteins. e.g.,

- Gene expression measurements
- Phylogenetic profiles
- Location of proteins/enzymes in the cell

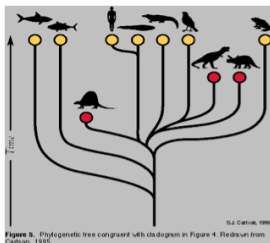
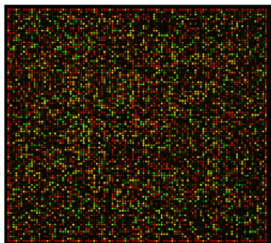
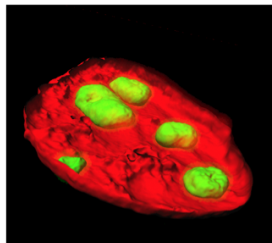


Figure 5. Phylogenetic tree congruent with cladogram in Figure 4. Redrawn from Cairns, 1995.



# Problem 1 : how to infer relationships between genes from biological data?

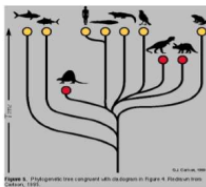
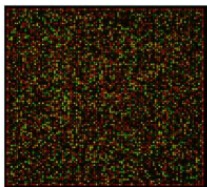
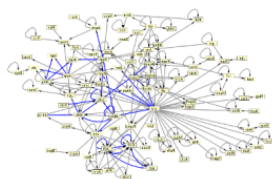
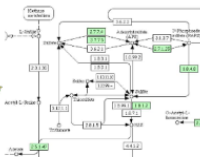
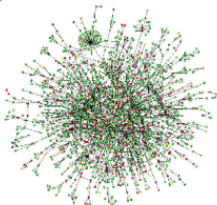


Figure 4. Phylogenetic tree compared with its diagram in Figure 4. *Phylogenetic tree*, DeSmet, 1999.



Inference



# Problem 2 : how to use biological networks to help in the analysis of genomic data?

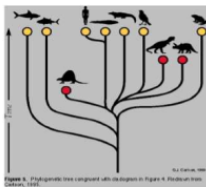
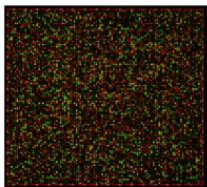
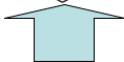
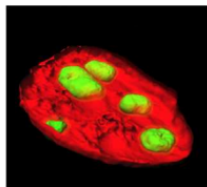
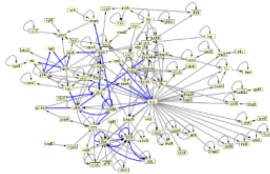
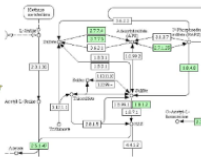
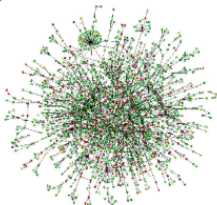


Figure 6. Phylogenetic tree compared with its diagram in Figure 4. (Rohlf and Bookstein, 1999)



Interpretation

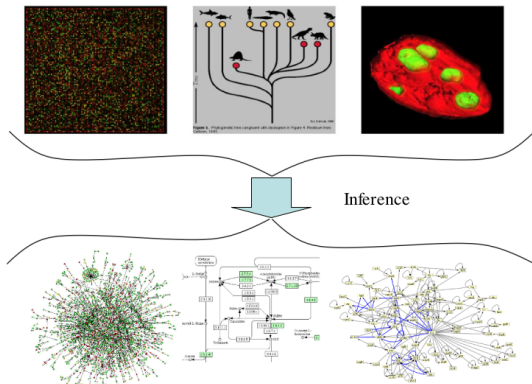


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- 2 How to use biological networks to help in the analysis of genomic data?

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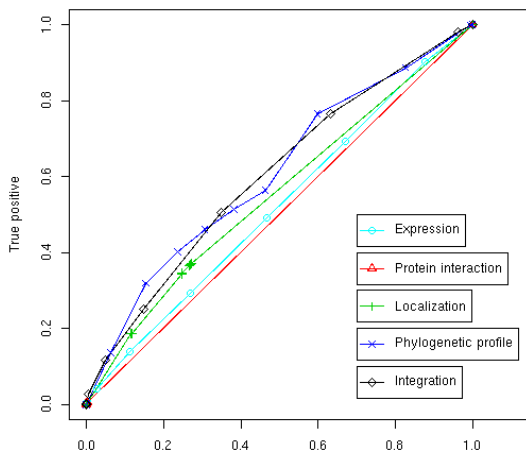
# Typical reverse engineering strategies

- Fit a **dynamical system** to time series (e.g., PDE, boolean networks, state-space models)
- Detect **statistical conditional independence or dependency** (Bayesian network, mutual information networks, co-expression networks, ...)



# Does it work? Case of metabolic network

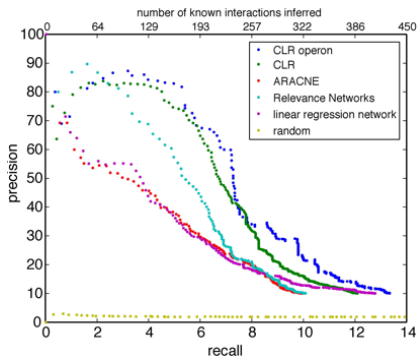
- The known metabolic network of the yeast involves **769 proteins**.
- Predict edges from distances between a variety of genomic data (expression, localization, phylogenetic profiles, interactions).





## Large-Scale Mapping and Validation of *Escherichia coli* Transcriptional Regulation from a Compendium of Expression Profiles

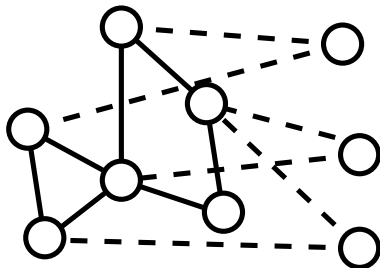
Jeremiah J. Faith<sup>1</sup>, Boris Hayete<sup>1</sup>, Joshua T. Thaden<sup>2,3</sup>, Ilaria Mogno<sup>2,4</sup>, Jamey Wierzbowski<sup>2,5</sup>, Guillaume Cottarel<sup>2,5</sup>, Simon Kasif<sup>1,2</sup>, James J. Collins<sup>1,2</sup>, Timothy S. Gardner<sup>1,2\*</sup>



## Motivation

In actual applications,

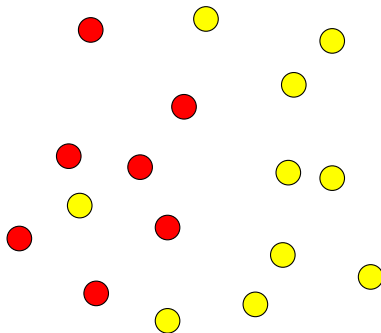
- we know in advance parts of the network to be inferred
- the problem is to add/remove nodes and edges using genomic data as side information



## Supervised method

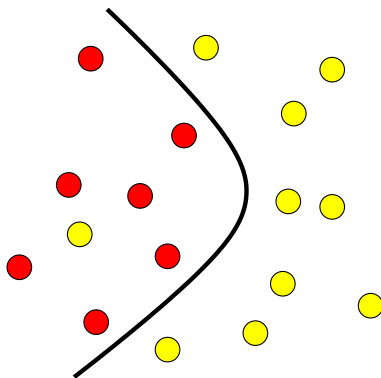
- Given genomic data **and** the currently known network...
- Infer **missing edges** between current nodes and additional nodes.

# Interlude : Pattern recognition



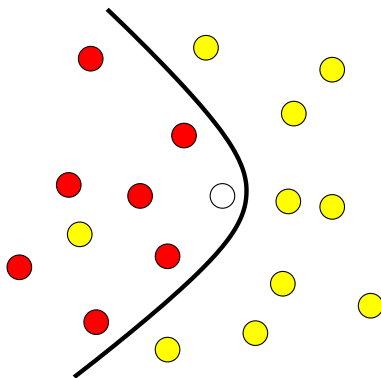
- Given a training set of patterns in two classes, learn to discriminate them
- Many algorithms (ANN, SVM, Decision trees, ...)

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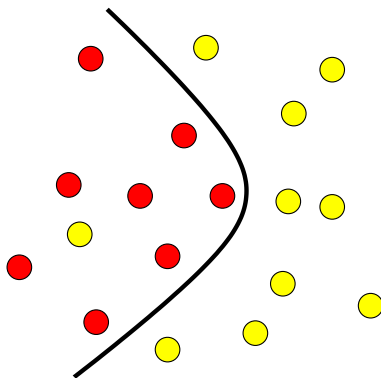
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# Pattern recognition and graph inference

## Pattern recognition

Associate a binary label  $Y$  to each data  $X$

## Graph inference

Associate a binary label  $Y$  to each **pair** of data  $(X_1, X_2)$

## Two solutions

- Consider each pair  $(X_1, X_2)$  as a single data -> **learning over pairs**
- Reformulate the graph inference problem as a pattern recognition problem at the level of individual vertices -> **local models**

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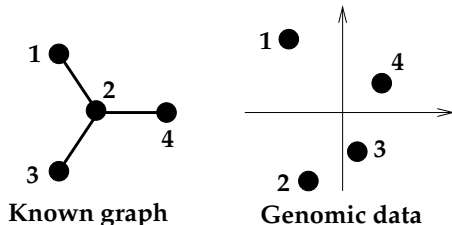
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# Pattern recognition for pairs

## Formulation and basic issue

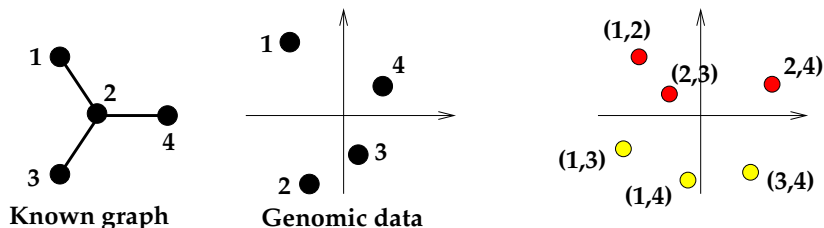
- A pair can be **connected (1)** or **not connected (-1)**
- From the known subgraph we can **extract examples** of connected and non-connected pairs
- However the genomic data characterize **individual** proteins; we need to work with **pairs** of proteins instead!



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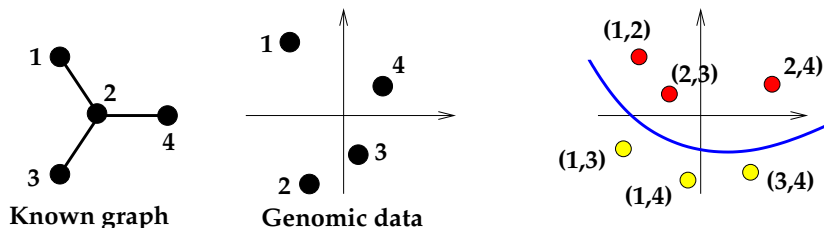
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## Concatenation?

- A simple idea is to **concatenate** the vectors  $u$  and  $v$  to obtain a  $2p$ -dimensional vector of  $(u, v)$ :

$$\psi(u, v) = u \oplus v = \begin{pmatrix} u \\ v \end{pmatrix}.$$

- **Problem:** a linear function then becomes **additive**...

$$f(u, v) = w^\top \psi(u, v) = w_1^\top u + w^\top v.$$

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## Symmetric tensor product (Ben-Hur and Noble, 2006)

$$\psi(u, v) = (u \otimes v) + (v \otimes u) .$$

**Intuition:** a pair  $(A, B)$  is similar to a pair  $(C, D)$  if:

- $A$  is similar to  $C$  **and**  $B$  is similar to  $D$ , **or**...
- $A$  is similar to  $D$  **and**  $B$  is similar to  $C$

## Metric learning (V. et al, 2007)

$$\psi(u, v) = (u - v)^{\otimes 2} .$$

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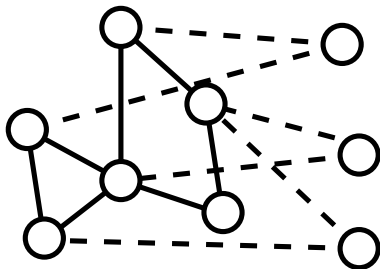
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# Supervised inference with local models

## The idea (Bleakley et al., 2007)

- Motivation: define **specific models** for **each target node** to discriminate between its neighbors and the others
- Treat each node independently from the other. Then **combine** predictions for ranking candidate edges.

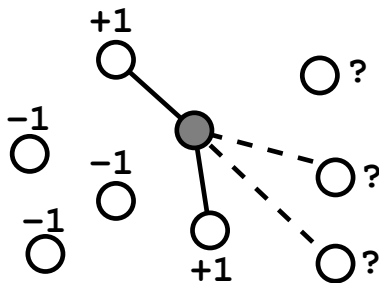
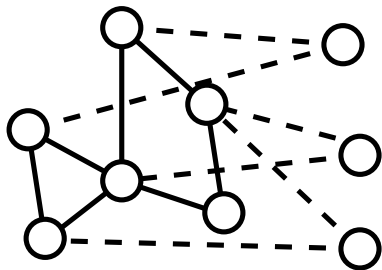




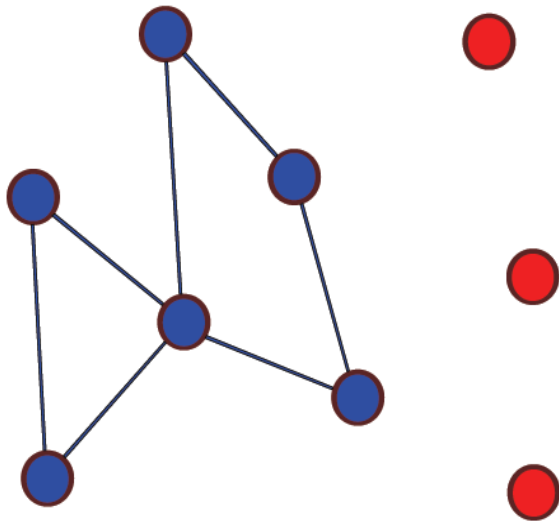
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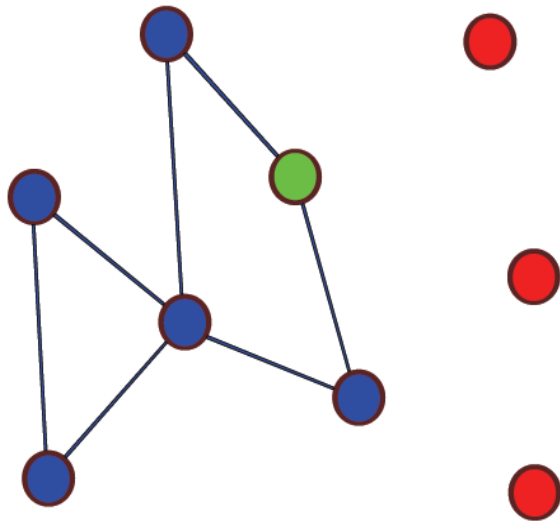
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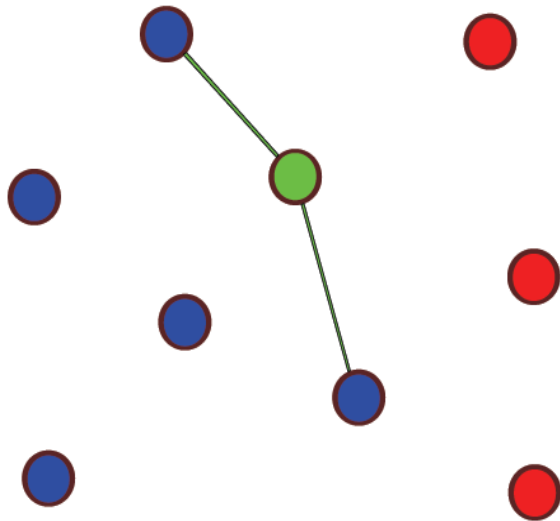
# The LOCAL model



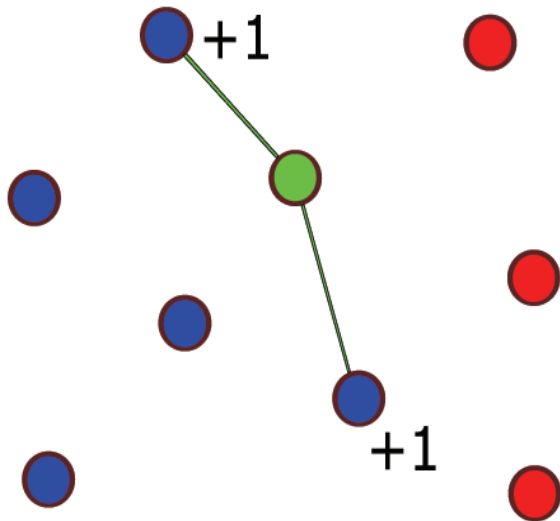
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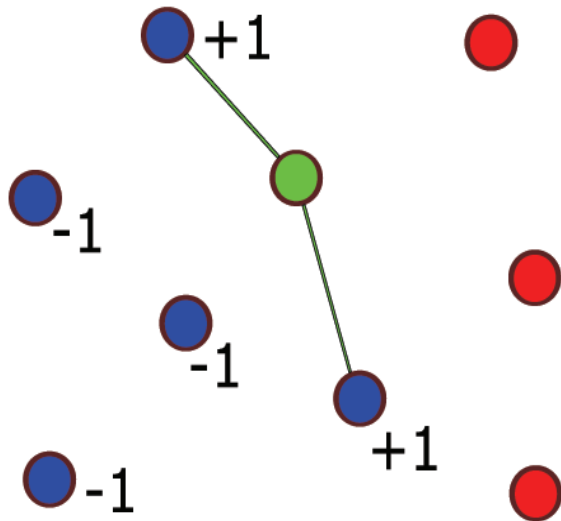
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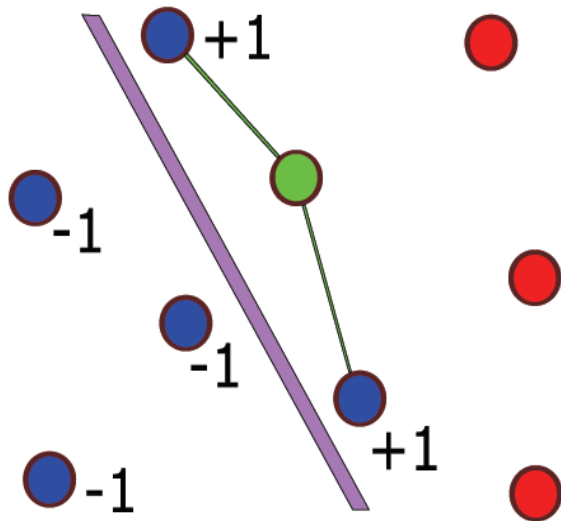
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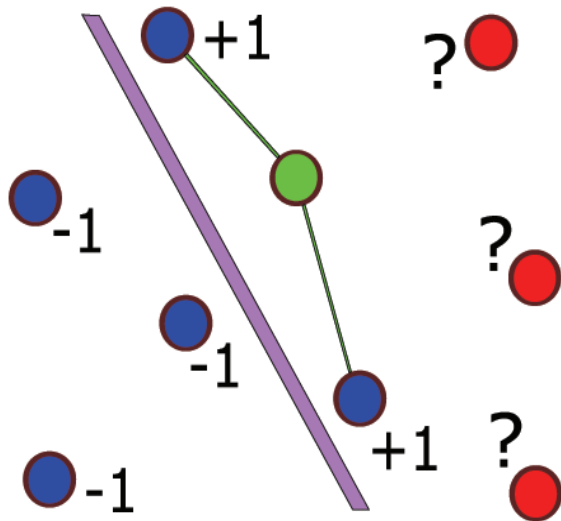
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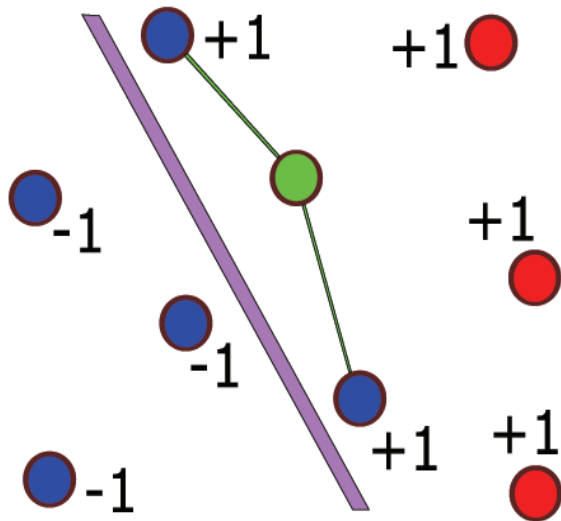


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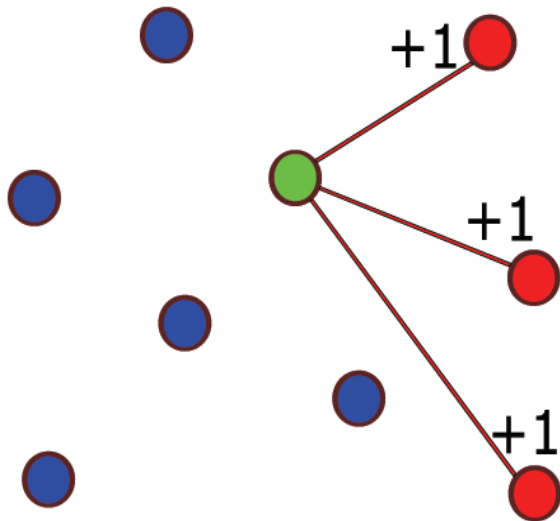




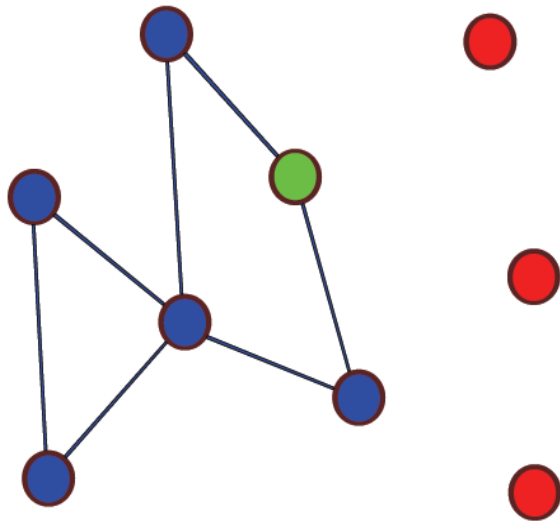
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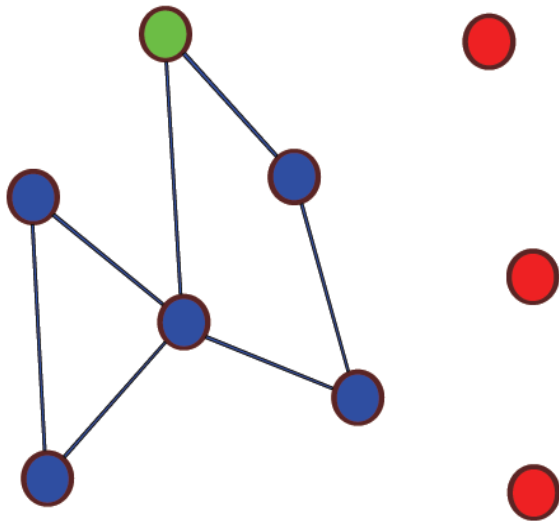
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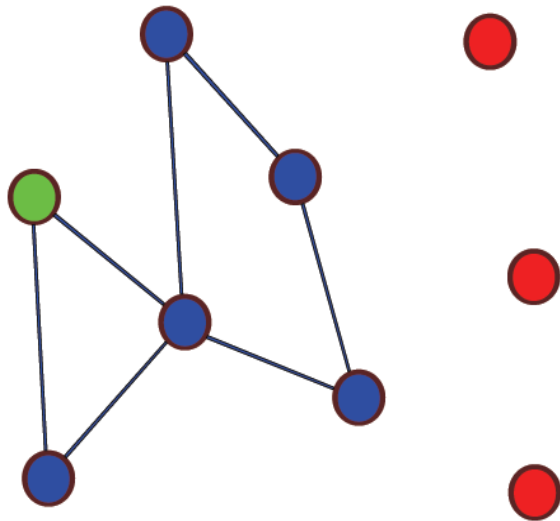
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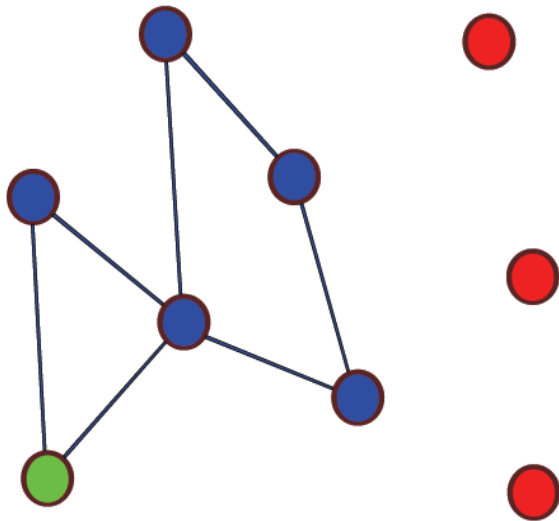
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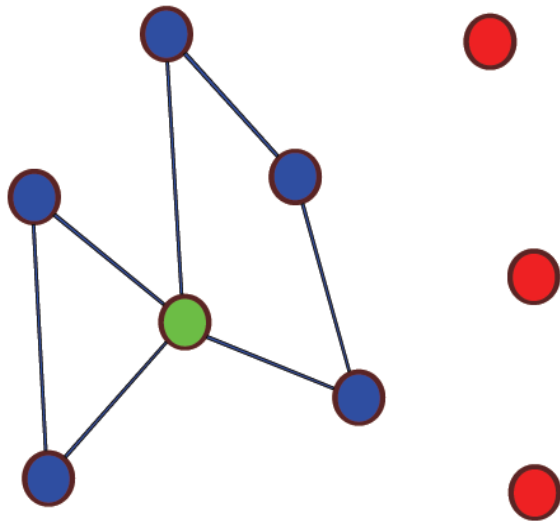
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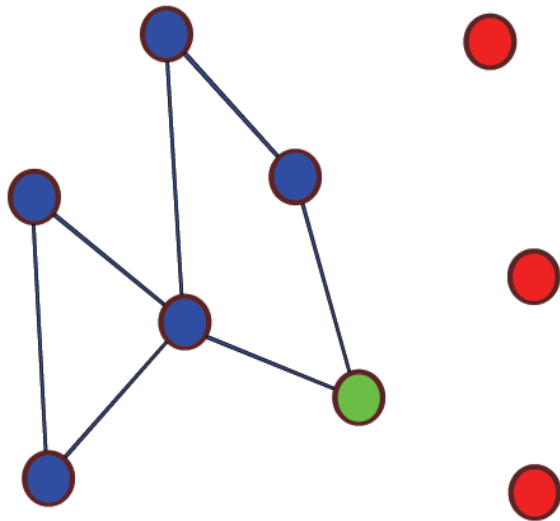
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# A few remarks about the local approach

- **Weak hypothesis:**
  - if A is connected to B,
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  - then A is likely to be connected to C.
- **Computationally:** much faster to train  $N$  local models with  $N$  training points each, than to train 1 model with  $N^2$  training points.
- **Caveats:**
  - each local model may have very few training points
  - no sharing of information between different local models

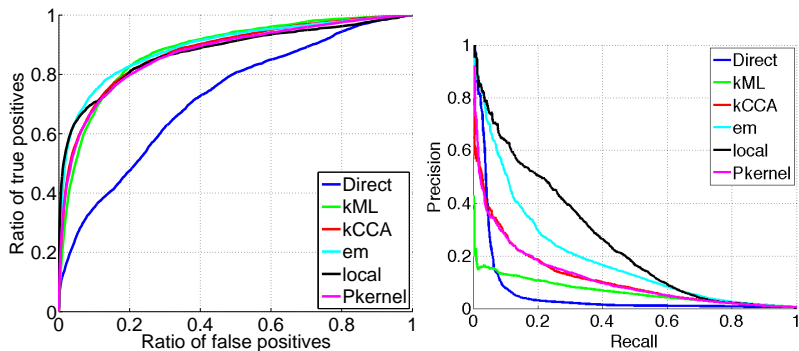
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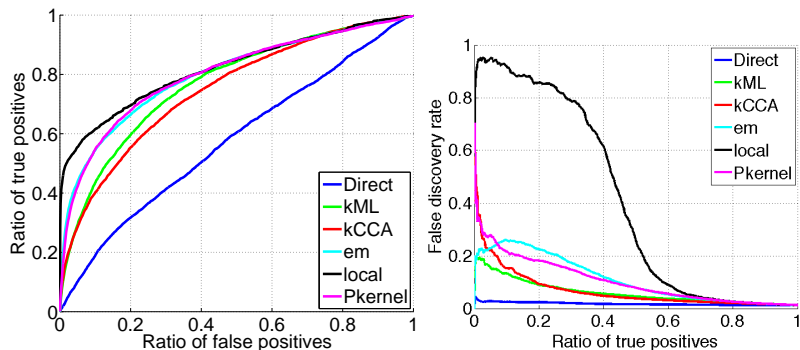
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# Results: protein-protein interaction (yeast)



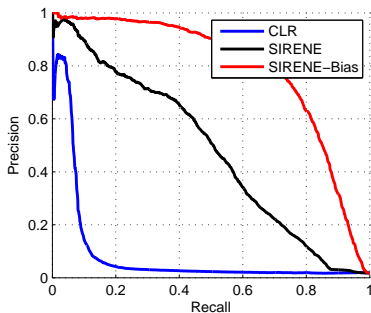
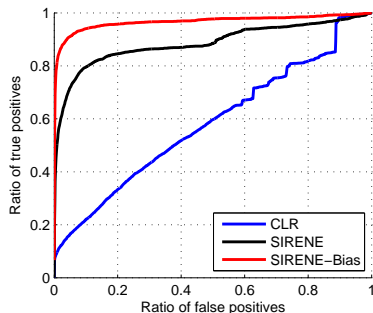
(from Bleakley et al., 2007)

# Results: metabolic gene network (yeast)



(from Bleakley et al., 2007)

# Results: regulatory network (E. coli)



Method	Recall at 60%	Recall at 80%
SIRENE	<b>44.5%</b>	<b>17.6%</b>
CLR	7.5%	5.5%
Relevance networks	4.7%	3.3%
ARACNe	1%	0%
Bayesian network	1%	0%

SIRENE = Supervised Inference of REgulatory NETworks (Mordelet and V., 2008)

## Prediction of missing enzyme genes in a bacterial metabolic network

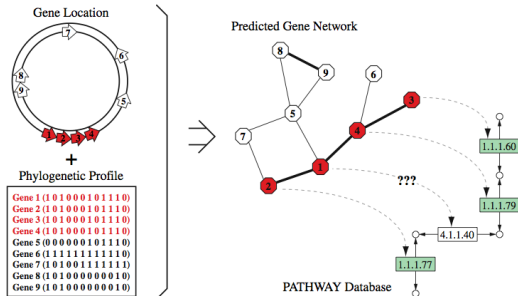
### Reconstruction of the lysine-degradation pathway of *Pseudomonas aeruginosa*

Yoshihiro Yamanishi<sup>1</sup>, Hisaaki Mihara<sup>2</sup>, Motoharu Osaki<sup>2</sup>, Hisashi Muramatsu<sup>3</sup>, Nobuyoshi Esaki<sup>2</sup>, Tetsuya Sato<sup>1</sup>, Yoshiyuki Hizukuri<sup>1</sup>, Susumu Goto<sup>1</sup> and Minoru Kanehisa<sup>1</sup>

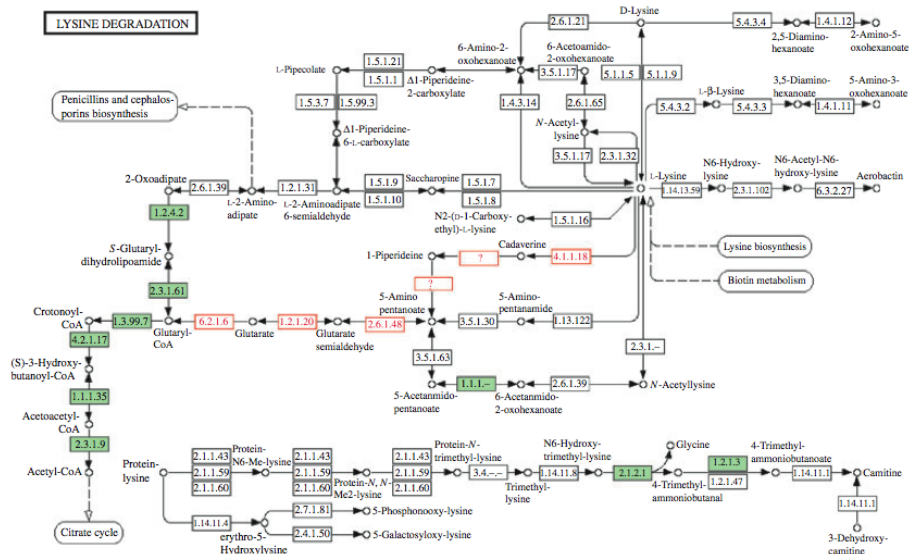
<sup>1</sup> Bioinformatics Center, Institute for Chemical Research, Kyoto University, Japan

<sup>2</sup> Division of Environmental Chemistry, Institute for Chemical Research, Kyoto University, Japan

<sup>3</sup> Department of Biology, Graduate School of Science, Osaka University, Japan



# Applications: missing enzyme prediction





## RESEARCH ARTICLE

## Prediction of nitrogen metabolism-related genes in *Anabaena* by kernel-based network analysis

*Shinobu Okamoto*<sup>1\*</sup>, *Yoshihiro Yamanishi*<sup>1</sup>, *Shigeki Ehira*<sup>2</sup>, *Shuichi Kawashima*<sup>3</sup>,  
*Koichiro Tonomura*<sup>1\*\*</sup> and *Minoru Kanehisa*<sup>1</sup>

<sup>1</sup> Bioinformatics Center, Institute for Chemical Research, Kyoto University, Uji, Japan

<sup>2</sup> Department of Biochemistry and Molecular Biology, Faculty of Science, Saitama University, Saitama, Japan

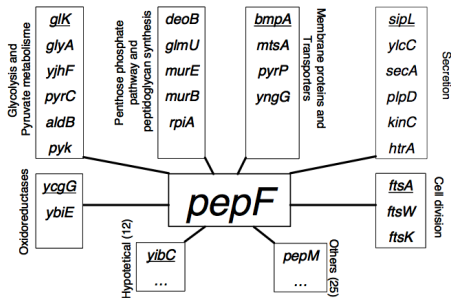
<sup>3</sup> Human Genome Center, Institute of Medical Science, University of Tokyo, Meguro, Japan

## Determination of the role of the bacterial peptidase PepF by statistical inference and further experimental validation

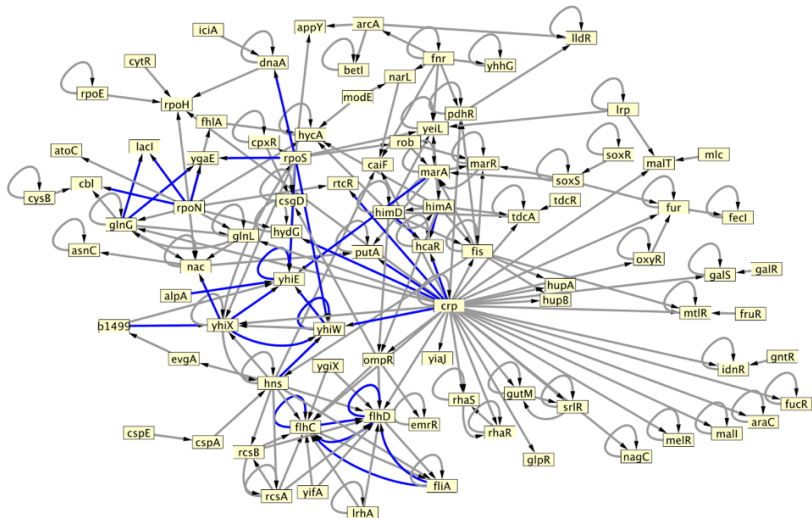
Liliana LOPEZ KLEINE<sup>1,2</sup>, Alain TRUBUIL<sup>1</sup>, Véronique MONNET<sup>2</sup>

<sup>1</sup>Unité de Mathématiques et Informatiques Appliquées. INRA Jouy en Josas 78352, France.

<sup>2</sup>Unité de Biochimie Bactérienne. INRA Jouy en Josas 78352, France.



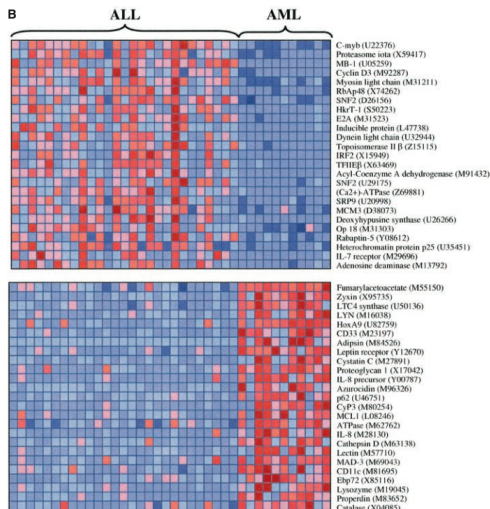
# Application: predicted regulatory network (E. coli)



Prediction at 60% precision, restricted to transcription factors (from Mordelet and V., 2008).

- 1 How to infer relationships between genes from biological data?
- 2 How to use biological networks to help in the analysis of genomic data?

# Tissue classification from microarray data



## Goal

- Design a **classifier** to automatically assign a class to future samples from their expression profile
- **Interpret** biologically the differences between the classes

## Issue

20K+ genes but only <100 tumours

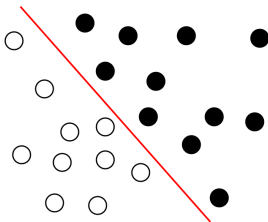
# Linear classifiers and signatures

## The model

- Each sample is represented by a vector  $x = (x_1, \dots, x_p)$
- **Goal**: estimate a linear function:

$$f_{\beta}(x) = \sum_{i=1}^p \beta_i x_i + \beta_0 .$$

- **Interpretability**: the weight  $\beta_i$  quantifies the influence of feature  $i$  (but...)



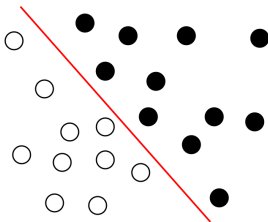
## Training the model

- Minimize an **empirical risk** on the training samples:

$$\min_{\beta \in \mathbb{R}^{p+1}} R_{emp}(\beta) = \frac{1}{n} \sum_{i=1}^n l(f_{\beta}(x_i), y_i),$$

- ... subject to some **constraint** on  $\beta$ , e.g.:

$$\Omega(\beta) \leq C.$$



- **Feature selection** (NP-hard, many greedy variants exist):

$$\Omega_{\text{Best subset selection}}(\beta) = \|\beta\|_0 = \sum_{i=1}^p \mathbf{1}(\beta_i > 0).$$

- **Small weights** (SVM, ridge regression, ...):

$$\Omega_{\text{ridge}}(\beta) = \|\beta\|_2^2 = \sum_{i=1}^p \beta_i^2.$$

- **Sparsity-inducing convex priors** (computationally tractable + feature selection):

$$\Omega_{\text{LASSO}}(\beta) = \|\beta\|_1 = \sum_{i=1}^p |\beta_i|.$$



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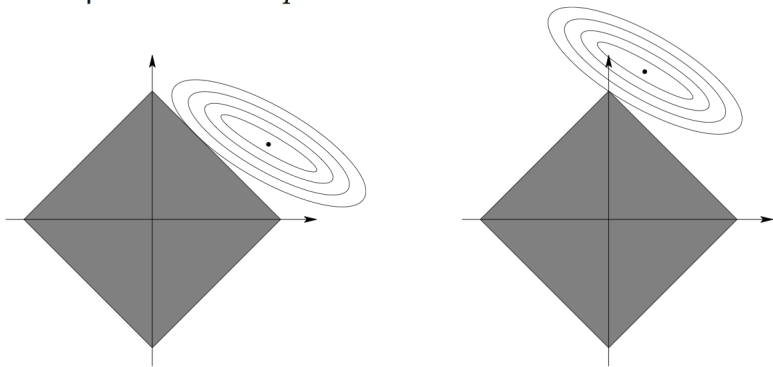
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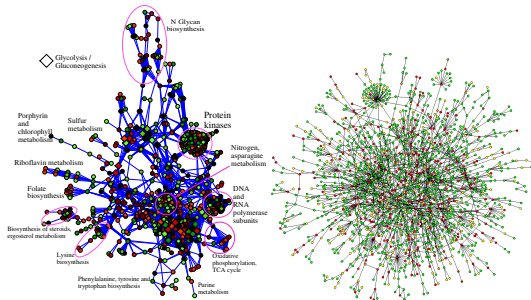
# Why LASSO leads to sparse solutions

Geometric interpretation with  $p = 2$



# How protein networks can help us

- Basic biological functions usually involve the **coordinated action of several proteins**:
  - Formation of **protein complexes**
  - Activation of metabolic, signalling or regulatory **pathways**
- Many pathways and protein-protein interactions are **already known**
- **Hypothesis**: the signature should be “coherent” with respect to this **prior knowledge**

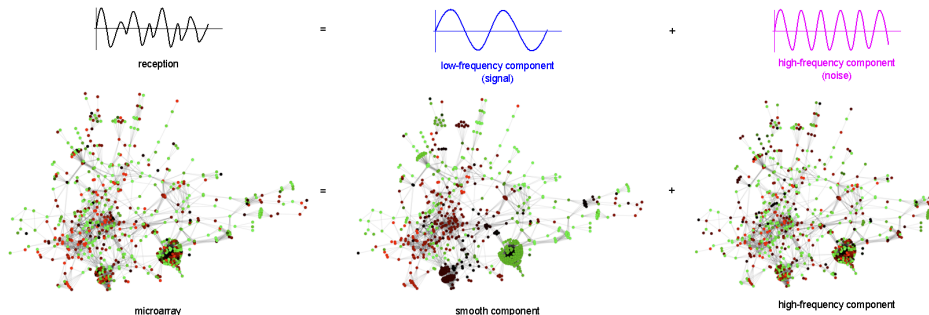


# Example: smooth signature

- Hypothesis: **adjacent genes** should have **similar weights** in the signature
- Penalty function (Rapaport et al., 2007):

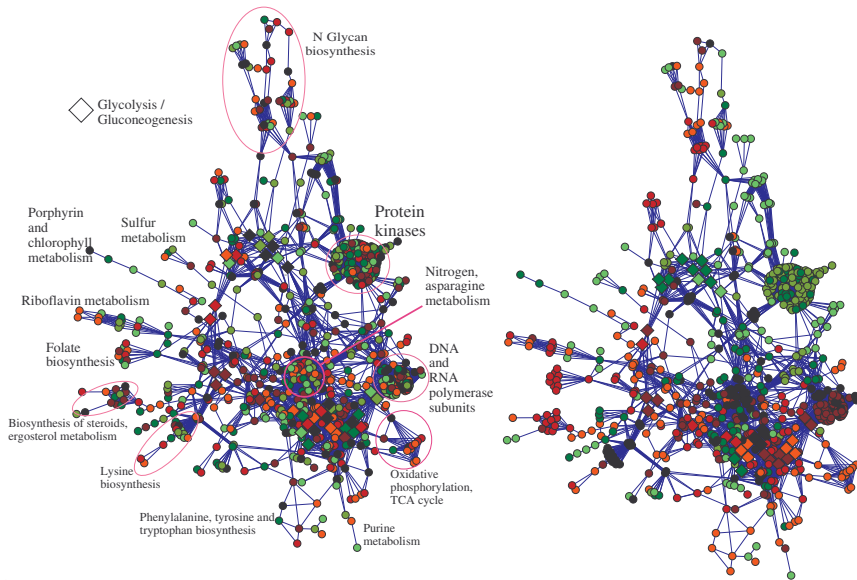
$$\Omega_{smooth}(\beta) = \sum_{i \sim j} (\beta_i - \beta_j)^2$$

# Equivalent formulation

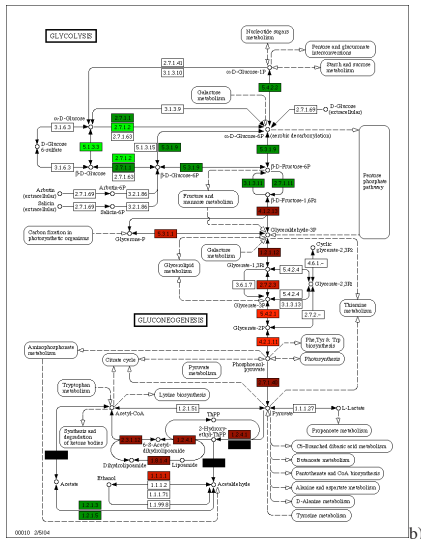
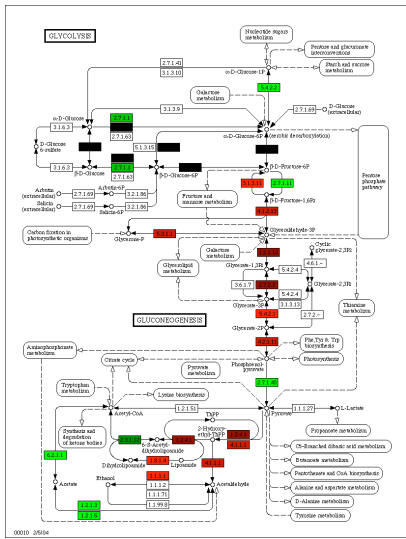


- 1 Use the gene network to extract the “important information” in gene expression profiles by **Fourier analysis** on the graph
- 2 Learn a linear classifier on the **smooth components** with classical ridge penalty.

# Illustration (yeast, high vs. low irradiation doses)



# Signatures



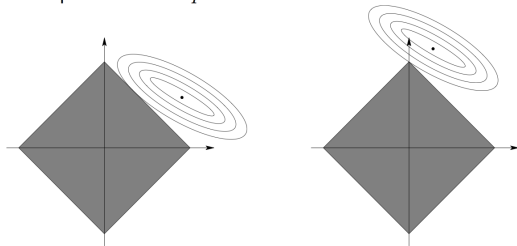


# Example: smooth and sparse signature

- Hypothesis:
  - the signature should be **sparse** (gene selection)
  - **connected genes** should have the **same weight**
- Penalty function (Rapaport et al., 2008):

$$\Omega_{\text{piecewiseconstant}}(\beta) = \sum_{i \sim j} |\beta_i - \beta_j| + \lambda \sum_i |\beta_i|.$$

Geometric interpretation with  $p = 2$



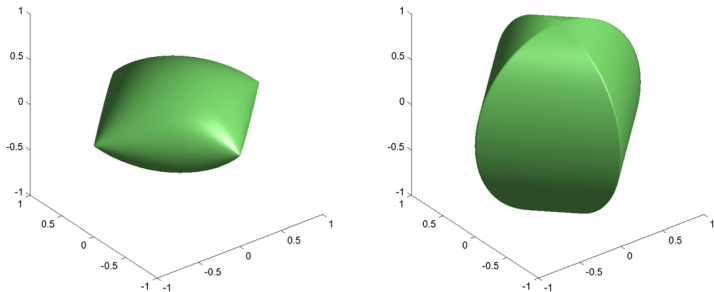
# Example: sparse pathway signature

- Hypothesis:
  - the signature should be **sparse** (gene selection)
  - selected genes should form **dense connected components** (without any constraint of their relative weights)
- Penalty function (Jacob et al., 2009):

$$\Omega_{intersection}(\beta) = \sum_{i \sim j} \sqrt{\beta_i^2 + \beta_j^2},$$

$$\Omega_{union}(\beta) = \sup_{\alpha \in \mathbb{R}^p: \forall i \sim j, \|\alpha_i^2 + \alpha_j^2\| \leq 1} \alpha^T \beta.$$

# Graph LASSO leads to structured sparsity



*Groups (1, 2) and (2, 3). Left:  $\Omega_{\text{intersection}}(\beta)$ . Right:  $\Omega_{\text{union}}(\beta)$ . Vertical axis is  $\beta_2$ .*

## Breast cancer data

- Gene expression data for 8,141 genes in 295 breast cancer tumors.
- Canonical pathways from MSigDB containing 639 groups of genes, 637 of which involve genes from our study.

METHOD	$\ell_1$	$\Omega_{\text{OVERLAP}}^G(\cdot)$
ERROR	$0.38 \pm 0.04$	$0.36 \pm 0.03$
# PATH.	148, 58, 183	6, 5, 78
PROP. PATH.	0.32, 0.14, 0.41	0.01, 0.01, 0.17

- Graph on the genes.

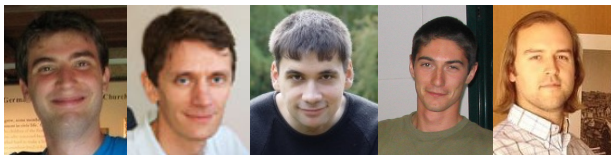
METHOD	$\ell_1$	$\Omega_{\text{graph}}(\cdot)$
ERROR	$0.39 \pm 0.04$	$0.36 \pm 0.01$
AV. SIZE C.C.	1.1, 1, 1.0	1.3, 1.4, 1.2

- A supervised machine learning formulation leads to promising results on the problem of inferring unknown relationships between genes and proteins.
- Conversely, biological networks can help fighting the curse of dimensionality for classification of high-dimensional genomic data
- All this is progressing very quickly these days!

# People I need to thank



- **Graph inference** : Yoshihiro Yamanishi, Minoru Kanehisa (Univ. Kyoto), Jian Qian, Bill Noble (Univ. Washington), Kevin Bleakley, Gerard Biau (Univ. Montpellier), Fantine Mordelet (ParisTech)
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